Anomalous Contraction Scour?
Vertical-Contraction Case

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Abstract: Clear-water scour due to a short vertical (pressure flow) contraction was investigated in a laboratory channel. Two approach velocities were studied for a (nominally) single configuration of depth and degree of contraction, with experiments conducted for various durations up to a maximum of 48 h, and the evolution of the scour hole over time monitored. The location of maximum scour in both cases was observed to occur downstream of the contraction, with maximum scour depths substantially in excess of values predicted by published models, even though equilibrium scour conditions were not reached.

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Introduction and Review

Recent research on bridge scour has focused on local scour, such as scour around bridge piers (e.g., Melville and Sutherland 1988; Melville and Chiew 1999) or near abutments (e.g., Oliveto and Hager 2002; Coleman et al. 2003); by comparison, contraction scour has received much less attention. The current work examines vertical-contraction or pressure-flow scour, which may arise when the water surface elevation upstream of the bridge rises above the bridge low chord, thus forcing a flow contraction, usually leading to a “pressure-flow” condition (see Fig. 1 for a definition sketch). As a result, scour may be induced, with the practical parameter of interest being the (equilibrium) maximum scour depth $Z_{eq}$. The present work was motivated largely by the study reported by Umbrell et al. (1998), to be referred to as UYSJ98, who performed experiments at constant upstream flow depth, $H_{up}$, for various median sediment sizes $d_{50}$, approach velocities $V_{up}$, and bridge-opening ratios, $H_{50}/H_{up}$, where $H_{50}$ is the initial (prior to scour) vertical bridge opening. A similar investigation by Arneson (1997), to be referred to as A97, also examined the effect of $H_{up}$ and included some live-bed (i.e., with upstream sediment transport) cases.

The long time scales associated with scour, particularly clear-water scour, has only been recently emphasized. In pier scour, Melville and Chiew (1999) observed that, while most of scour does occur early in the process, less than 50% of the ultimate equilibrium scour was achieved within 10–12 h, and Oliveto and Hager (2002) suggested that the concept of ultimate equilibrium scour may be inadequate, and that the scour depth may grow continually (though in a slow logarithmic fashion) over time without necessarily asymptoting to any ultimate value. The duration $T$ of the experiments of UYSJ98 was however quite short, approximately 3.5 h, but a somewhat ad hoc extrapolation, based on pier scour, to equilibrium conditions was applied. In A97, it was stated that the experiments were continued until equilibrium scour was achieved, but specific durations were not given.

The present work reports on an experimental study of clearwater (i.e., no upstream sediment transport) vertical-contraction scour. It differs from UYSJ98 and A97 in examining in more detail features of the scour hole and their time development. Hence, only a single geometric configuration with a single uniform sand was investigated, though two approach velocities were studied over a range of durations.

Experimental Equipment, Materials, and Procedure

The experiments were carried out in the Purdue Hydrodynamics Laboratory with an 11-m long flume that is 0.45-m wide and 0.72-m high. The test section was located approximately 3.2 m from the flume inlet. The test sediment was a uniform medium Ottawa sand ($d_{50}=0.6$ mm, and geometric standard deviation $\sigma_g=1.15$). The critical shear velocity $(U_{cr})$ was estimated from the traditional Shields diagram to be 1.79 cm/s, corresponding to a critical Shields parameter, $\theta_{cr}=(U_{cr})^2/(g(s-1)/d_5)=0.032$, where $g$ is the acceleration due to gravity and $s=2.65$ is the sediment specific gravity. A critical upstream velocity $(V_{up})_{crit}$ of 35 cm/s was estimated following the procedure in A97. A point gauge was used for measuring flow depths and bed elevations with an uncertainty in bed elevation measurement estimated to be 0.5 mm, due to operator judgment as to where the bed was located relative to the point.

Two series of clear-water experiments were carried out with $H_{up}=10.5$ cm but different $V_{up}=22.8$ and 25.6 cm/s. Each series consisted of different runs, and so $V_{up}$ varied to a limited extent between runs. In contrast to both UYSJ98 and A97, the modeled “bridge” was of simple rectangular form [Fig. 1(b)], without a realistic substructure, e.g., bottom girders [Fig. 1(a)]. The proportions are similar to those of UYSJ98 with a length-to-height ($L_b/h_b$) ratio of $\approx 7$, but the chosen sizes ($L_b=15.3$ cm and $h_b \approx 2.2$ cm) were smaller by a factor of 4–5. Only a single geometric configuration was considered, namely, one with a moderate blockage when the bridge model was almost completely...
submerged but there was no overtopping, with \( H_{bd}/H_{ap} \approx 0.78 \).

The range of current experimental conditions is compared with those of UYS98 and A97 in Table 1.

Although ostensibly clear-water conditions \([V_{ap}/(V_{ap})_{crit} < 0.75]\) prevailed upstream of the model bridge, small-scale bed forms or ripples developed near the flume inlet where the surface bed material changed from gravel to medium sand. In order to minimize the extraneous effects of these ripples on the approach flow, a run would be terminated when the ripples arrived within 0.5 m of the model bridge. The time available before this would occur depended sensitively on 0.5 m of the model bridge. The time available before this would occur depended sensitively on \( V_{ap} \), which limited severely the range of \( V_{ap} \) and/or \( T \). For the lower-velocity case, ripples were essentially absent throughout the inlet and approach region during the first 24 h, but after 48 h, they were observed at a distance of 1-m upstream of the model bridge. For the higher-velocity case, ripples were already noted after 3 h, and the experiment had to be terminated at \( T = 18 \) h. For the lower-velocity case, five different \( T = 6–48 \) h were studied, while for the higher-velocity case, only three different \( T \) could be studied. Each run was independent in that the bed was reformed after each run so that each run began with the same unscoured bed. For convenience, the various runs will be labeled according to whether the approach velocity was high or low, and the nominal duration in hours. In cases, where multiple runs were done for the same conditions, the different runs are distinguished by an alphabetical suffix. Thus, Low36A refers to the first 36-h run at the lower approach velocity.

Before an experimental run, the test bed was levellied with a screed mounted on the mobile carriage, so that the initial state of the bed was nominally the same for each run. Flow was then started at a slow rate to inhibit any initial sediment transport, and the pump-drive frequency and downstream gate opening were then adjusted to obtain the desired flow depth and discharge. This flow was allowed to attain a quasi-equilibrium for approximately 30 min before the model bridge was introduced from above into the flow. The time instant at which the model bridge was completely installed was taken as the time origin \((t=0)\). The flow was frequently checked to ensure that the approach flow depth was maintained. After the water had drained, usually overnight, from the channel, a survey of the bed was taken with the point gauge, with bed elevations being measured along five parallel lines in the streamwise direction.

### Results

The coordinate system is chosen with origin at the initial bed elevation on the channel centerline where the bridge/obstruction begins [see Fig. 1(b)]. Horizontal coordinates, \( x \) and \( y \), are non-dimensionalized by the length of the bridge (in the streamwise direction) \( L_b \), such that \( x/L_b \) and \( y \) designate the streamwise and vertical coordinates, respectively. Although various parameters were measured, only the depth of scour \( z_b \) and the horizontal distance \( x_b \) from the front of the model bridge were employed for the subsequent comparison of results.

For the lower-velocity case, some scour began to be noticeable only after 30–45 min, with the appearance of ripplelike features after an hour. Photographs at various time instants during run Low36A are shown in Fig. 2. After \( \approx 2–3 \) h, a sizeable scour hole was already noticeable downstream of the model bridge [Fig. 2(b)]. The occurrence of the major scour downstream of the contracted section \((x/L_b > 1)\) is unexpected; from a traditional hydraulic analysis, the maximum velocity and bed shear stress, and hence maximum scour would be expected to occur within the contraction. The bed also retained to a high degree its lateral uniformity for all runs, with rather mild lateral features becoming noticeable only at long times \((T \approx 36 \) h\); three-dimensional effects did not seem to have played any significant role for the lower-velocity series. In contrast, the bed in the higher-velocity case showed signs of scour very soon (within minutes) after the introduction of the bridge, and lateral features also became noticeable at much earlier times.

### Scour-Hole Geometry over Time

The time development of the scour hole is shown in Fig. 3, where streamwise profiles of bed elevations \( z_b \) for various values of \( T \) are plotted. The profiles shown were all taken at \( y/L_b \approx 0.5 \), since the maximum measured scour depth \( Z_m \) generally occurred along this line (for all durations except the 6-h duration). In both series of experiments, one duration was repeated at least twice, which allowed a check on the reproducibility of the scour results. In the lower-velocity series, the 36-h run was repeated, and the degree of reproducibility was quite high. The profiles from the three repeated 12-h runs in the higher-velocity series exhibit more variability. Whereas the profile up to the location of maximum scour \( x_m/L_b \approx 0.5 \), the geometry of the region downstream of \( x_m/L_b \) can differ by as much as 80% of \( Z_m \). This lack of reproducibility may be related to the cyclic phenomena reported in plane-wall-jet scour study of Balachandar and Kells (1997), in which bed profiles downstream of the scour trough could change significantly over a short time period.

For the lower-velocity series, the scour hole develops in a very regular manner with \( Z_m \) occurring downstream of the contracted region \((x_m/L_b > 1)\) for all \( T \). Even after 48 h, scour still seems to be occurring, and an equilibrium scour depth still has not yet been achieved. Downstream and particularly upstream of the trough however, the local slopes \([1 \approx 3.3 \text{ upstream and } 1 \approx 6.7 \text{ downstream}] \)

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**Table 1. Summary and Comparison of Experimental Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present</th>
<th>UYS98</th>
<th>A97</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{so} ) (mm)</td>
<td>0.6</td>
<td>0.3–2.4</td>
<td>0.6–3.3</td>
</tr>
<tr>
<td>((V_{ap})_{crit} ) (cm/s)</td>
<td>35</td>
<td>34–67</td>
<td>44–81</td>
</tr>
<tr>
<td>( V_{ap} ) (cm/s)</td>
<td>23–26</td>
<td>22–64</td>
<td>12–70</td>
</tr>
<tr>
<td>( H_{ap} ) (cm)</td>
<td>10.4–10.6</td>
<td>30.5</td>
<td>36–70</td>
</tr>
<tr>
<td>( h_b ) (cm)</td>
<td>2.1–2.3</td>
<td>1.6–15.2</td>
<td>0–32</td>
</tr>
<tr>
<td>( L_b ) (cm)</td>
<td>15.3</td>
<td>61</td>
<td>90</td>
</tr>
<tr>
<td>( T ) (hr)</td>
<td>6–48</td>
<td>3.5</td>
<td>N/a</td>
</tr>
<tr>
<td>( V_{ap} ) (cm/s)</td>
<td>0.22–0.25</td>
<td>0.13–0.37</td>
<td>0.06–0.33</td>
</tr>
<tr>
<td>( R_{ap}(\times 10^3) )</td>
<td>22.8</td>
<td>67–194</td>
<td>52–340</td>
</tr>
<tr>
<td>( V_{ap} / (V_{ap})_{crit} )</td>
<td>0.64–0.73</td>
<td>0.44–1.10</td>
<td>0.22–1.49</td>
</tr>
<tr>
<td>( H_{ap} / d_{so} )</td>
<td>175</td>
<td>127–1016</td>
<td>126–925</td>
</tr>
<tr>
<td>( h_b / d_{so} )</td>
<td>36</td>
<td>6–510</td>
<td>10–510</td>
</tr>
</tbody>
</table>

For consistency, \((V_{ap})_{crit} \) is evaluated according to A97 in this table.
downstream; note that the slopes shown in Fig. 3 are physical slopes, i.e., \( \Delta z: \Delta x \) and not scaled slopes, i.e., not \( \Delta (z/L_b): \Delta (x/L_b) \) are approximately constant, suggesting that these regions are in approximate equilibrium. For comparison, a typical angle of repose (32°) corresponds to a slope of 1 in 1.6. The notable asymmetry in slopes points to quite different local flow and transport conditions in the regions upstream and the regions downstream of the trough, somewhat reminiscent of the shape of a sand dune. The upstream slope is however substantially smaller than that associated with a typical angle of repose, suggesting that the local flow there may be quite different from that just downstream of a dune crest.

The scour profiles observed in the higher-velocity series resembles those in the lower-velocity series in that the large bulk of the scour and \( Z_m \) still occur at \( x_m/L_b > 1 \), but significant scour also occurs under the model bridge, with noticeable scour already even at \( x/L_b = 0 \). Consequently, the scour-hole geometry is more complicated, with a shallower ledgelike region under the model bridge, followed immediately by a steeper trough region more similar to that seen in the lower-velocity series. This marked difference in scour geometry suggests different scour mechanisms in the region under the bridge and in the “scour hole” just downstream of the bridge. Because the longest experiment duration for the higher-velocity series was only 18 h, how close this scour-hole shape is to an “equilibrium” shape or \( Z_m \) is to \( Z_{me} \) is difficult to judge. Nevertheless, as in the lower-velocity run, the upstream profiles, in the vicinity of 0 < \( x/L_b < 0.5 \), do seem to asymptote to a common curve. The local slope (≈ 1 in 9.2) associated with this curve is of course rather smaller than that seen in the lower-velocity series.

**Time Scales and Evolution of \( Z_m \) and \( x_m \).** Various time scales have been suggested for scour problems. Oliveto and Hager (2002) defined a time scale, \( T_{OH} = L_{ref}/\sqrt{g(s-1)d_{so}} \), where \( L_{ref} \) is a characteristic reference length scale, such as a pier width. This yields an extremely small time scale on the order of seconds, which seems inappropriate for the long time scales associated with at least the later stages of scour-hole development; in addition, it does not depend on flow conditions. A simple scour time scale \( T \) may be motivated from a scaling argument. From a bal-
of the volume (per unit width) of sediment transported, 

\[ q_s T \sim L_z^2 \]

where \( q_s \) is a volumetric rate of sediment transport per unit width and \( L_z \) is an appropriate length scale characterizing the scour hole. Several traditional transport formulas can be expressed in a simplified form as 

\[ q_s = \frac{\theta \sqrt{g(s-1)d_{50}^2}}{2} \]

where \( \theta \) is a Shields parameter and \( n_s \) is a model constant (\( n_s = 3/2 \) in the Meyer-Peter-Müller model, and \( n_s = 5/2 \) in the Engelund-Hansen model). With the choice of \( L_z \sim h_b \), and with \( \theta = \theta_{up} \) being based on the upstream shear velocity \( (u_{up})_{up} \)

\[ T = \frac{h_b^2}{q_s} = \frac{h_b^2}{\theta (u_{up})_{up} \sqrt{g(s-1)d_{50}^2}} \]  

(1)

In the following, \( n_s \) was chosen for simplicity to be 2, which is in the expected range, and yields reasonable magnitudes. For the lower-velocity series \( T = 9.7 \) h; in comparison, \( n_s = 3/2 \) or 5/2 would yield respectively \( T = 1.2 \) (less plausible) or 80 h (also plausible). The choice of \( (u_{up})_{up} \) as an appropriate velocity scale may be debated, but is based partially on convenience, since it is readily available, as well as on physical considerations. A traditional one-dimensional viewpoint would argue that \( (u_{up})_{up} \) would remain relevant for \( x/L_b > 1 \), where the maximum scour is observed. A less naive argument might include the effect of contraction on \( \theta \) through a dependence on the degree of contraction, e.g., via \( H_{50}/H_{up} \), but for the moderate contraction under consideration, the effect is unlikely to be dramatic. The present work is not aimed at testing the general validity of this time scale, but rather only uses this in presenting dimensionless results.

The variation of both \( Z_m/h_b \) and \( x_m/L_b \) with nondimensional time \( t/T \) for the lower-velocity series is shown in Fig. 4. Because the scour-hole geometry in the higher-velocity series is quite complicated, and because of the larger uncertainty in the estimation of \( (u_{up})_{up} \), the results for that series are not shown. Fig. 4 indicates that \( Z_m/h_b \) does vary with time to a large degree in a logarithmic fashion over the range of durations studied. Oliveto and Hager (2002) proposed, in the context of local-scour problems, a correlation for the slope of the logarithmic variation, based also on a densimetric Froude number (and a specification of a reference length scale \( L_{rej} \)). With a \( L_{rej} \) according to their specification, the present slope \( = 1.5 \) is substantially larger (by more than a factor of 3) than their correlation.

The evidence for a logarithmic variation of the nondimensional location of maximum scour \( x_m/L_b \) with time (which would be equivalent to a linear relation between \( x_m \) and \( Z_m \) at least over some time period) is less clear because of greater scatter and the limited number of data points. A tentative fitted line, drawn in Fig. 4, is consistent with \( Z_m = (x_m/L_b)/3 - C h_b \), with \( C = 0.5 \). The coefficient of 1/3 is directly related to the 1 in 3.3 slope shown previously in Fig. 3. Although the logarithmic variation is unlikely to be uniformly valid for all time, and the dangers of extrapolation are recognized, a rough estimate of the location \( x_m \) where the scour started can be based on the fitted lines. These show that, as \( Z_m \to 0 \), then \( t/T \approx 1.6 \) h and \( x_m/L_b \approx 1.2 \), i.e., the scour begins downstream of the model bridge. Photograph evidence (Fig. 2) indicates that scour downstream of the model was already noticeable even early in the scour-hole development.

Model Predictions of \( Z_m \). The ultimate scour depth predicted by the models of UYSJ98 and A97 for the two cases studied are also shown in Fig. 3. These fall well short of the observed \( Z_m \) for both the low48 and high18 cases, even though an equilibrium has not yet been reached when these runs were terminated. In the case of UYSJ98, the discrepancy is attributed to the very short duration of their runs in spite of their ad hoc corrections. A97 stated only that experiments were run to equilibrium, and so whether duration brevity contributed to the observed discrepancy is not clear. As seen in Table 1, A97 did perform some experiments under conditions and parameter values similar to the present, but A97 also included live-bed experiments that might have contributed to a bias in that equation. In Lyn (2008), an alternative model equation was developed from a reanalysis of Arneson’s data for clear-water conditions, but it still underpredicted substantially the observed \( Z_m \). This points to a difference due not simply to methods of data analysis or to differences in sampling, but rather to differences in experiment design and procedure.

The present experimental configuration differs from that of A97 (and UYSJ98) in the absence of a model girder substructure, and so \( H_{50} \) is defined with respect to the bottom of the model bridge rather than to the bottom of a girder. The model girder and the alternative definition of \( H_{50} \) could, depending on the exact location of the girder, conceivably lead to reduced scour depths. Whether this could explain the over 100% difference in observed and predicted scour depths seems somewhat less plausible. A97 noted that bed forms were present in most of the experiments, but whether these were directly related to the scour process, or were only background bed forms is not clear. In the present work, background bed forms not specifically related to the scour process were by experimental design avoided.

The scale of the present study is notably smaller than that of both UYSJ98 (by a factor of 3–4) and A97 (by a factor of 4–7), and possible scale effects might be considered. In terms of the relevant (and some possibly not so relevant) dimensionless parameters, with the exception of the upstream Reynolds number \( R_{up} \), the parameters of the current study lie within the range of the other studies, though toward the lower end (Table 1). The parameter of greatest concern likely involves \( d_{50} \), such as \( h_b/d_{50} \), but it is believed that \( h_b/d_{50} \approx 36 \) is sufficiently large such that extraneous effects of sediment scaling are not important. For comparison, in bridge-pier scour, Melville and Sutherland (1988) and, in abutment scour, Coleman et al. (2003) argued in terms of pier or abutment length to sediment size ratio, which in the present context might be equivalent to \( h_b/d_{50} \). Both studies suggested that, for structure to sediment size ratios larger than 25, the effect of this ratio on scour depth was insignificant. While scale effects cannot be entirely excluded, their effects are unlikely to explain the different phenomena seen in the present study.

Fig. 4. Dimensionless time evolution of the maximum scour depth \( Z_m/h_b \) and the location of maximum scour \( x_m/L_b \) in log-linear coordinates—lines are the corresponding best-fit log-linear models.

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Summary and Conclusions

Measurements of scour-hole geometry were made for various experimental durations in a laboratory study of clear-water scour due to a vertical contraction. Only a single approach flow depth and degree of contraction, a single sediment size but two approach velocities were studied. The maximum scour for both approach velocities occurred at a location downstream of the contracted section; for the lower-velocity series but not for the higher-velocity series, scour was actually negligible within the contracted section. In the lower-velocity series, scour continued to occur, even 48 h after imposition of the contraction, with the maximum scour depth growing at a logarithmic rate over the range of durations studied, and upstream and downstream of the scour trough, the geometry of the scour hole seemed to have attained an approximate equilibrium with upstream local slopes that are noticeably smaller than that associated with the particle angle of repose. Further, although scour had not yet ceased at the longest durations and hence an equilibrium was not yet reached, the maximum scour depth significantly exceeded predictions of equilibrium scour by published models.

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Notation

The following symbols are used in this technical note:

- $d_{50}$ = median diameter of test sediment;
- $F_{up}$ = Froude number ($=V_{up}/\sqrt{gH_{up}}$);
- $H_{ob}$ = distance from the initial bed level to the obstruction (low chord);
- $h_{up}$ = upstream depth;
- $h_{b}$ = depth of submergence of obstruction;
- $g$ = acceleration due to gravity;
- $L_{b}$ = length of obstruction in the streamwise direction;
- $L$ and $L_{ref}$ = structure length scale and reference length scale;
- $q$ = discharge per unit width;
- $R_{up}$ = Reynolds number based on upstream depth and velocity;
- $s$ = specific gravity of sediment;
- $T$ = duration of experiment (after installation of contraction);

- $T$ and $T_{OH}$ = time scale of scour of current study and defined by Oliveto and Hager (2002);
- $t$ = time coordinate;
- $(u_{c})_{up}$ and $(u_{c})_{crit}$ = upstream shear velocity and critical shear velocity for incipient sediment motion;
- $V_{crit}$ and $(V_{up})_{crit}$ = general critical velocity associated with incipient sediment motion and critical velocity based on upstream depth;
- $V_{up}$ and $V_{b0}$ = upstream (approach) velocity and velocity at bridge section prior to scour;
- $x$ = coordinate in streamwise direction with zero at the upstream edge of bridge;
- $x_{m}$ = x-coordinate of section with maximum scour;
- $y$ = coordinate in the lateral direction;
- $Z_{m}$ and $Z_{max}$ = maximum depth of scour and ultimate (equilibrium) maximum depth of scour;
- $z$ = vertical coordinate with origin at the initial bed elevation;
- $z_{bed}$ = bed elevation;
- $\theta$ and $\theta_{crit}$ = Shields parameter and value of Shields parameter for incipient sediment motion; and
- $\sigma_g$ = geometric standard deviation of sediment size distribution.

References


